

Physics 1C Discussion (Week 7)

Problem 1

Three polarizing filters are stacked, with the polarizing axis of the second and third filters at 23.0° and 67.0° , respectively, clockwise to that of the first. If unpolarized light is incident on the stack, the light has intensity 4.60 W/cm^2 after it passes through the stack. If the incident intensity is kept constant but the second polarizer is removed, what is the intensity of the light after it has passed through the stack?

First let us calculate the final intensity with three polarizers. For the case with all three polarizers, the transmitted intensity after passing through each filter is calculated using Malus's law:

$$I_{\text{outgoing}} = I_{\text{incoming}} \cos^2(\theta)$$

where θ is the angle difference between successive polarizers.

For the unpolarized light passing through the first polarizer, you can first apply Malus's law, but you also need to average over whole possible polarization angle of the incoming light with initial intensity I_0 . Therefore, the effective factor is

$$\begin{aligned} I_1 &= I_0 \left[\frac{1}{2\pi} \int_0^{2\pi} d\theta \cos^2(\theta) \right] \\ &= \frac{1}{2} I_0 \end{aligned} \quad (1)$$

Secondly, we consider the filtered light passing through the second polarizer. The angle between the first and second polarizer is 23.0° , then from Malus's law

$$I_2 = I_1 \cos^2(23.0^\circ) = \frac{I_0}{2} \cdot \cos^2(23.0^\circ). \quad (2)$$

For the light passing through the third polarizer, the angle difference between the second and third polarizers is $67.0^\circ - 23.0^\circ = 44.0^\circ$. Then

$$I_3 = I_2 \cos^2(44.0^\circ). \quad (3)$$

After passing through all three polarizers, we are given that the final intensity is 4.60 W/cm^2 . Thus

$$I_3 = 4.60 \text{ W/cm}^2 = \frac{I_0}{2} \cos^2(23.0^\circ) \cos^2(44.0^\circ). \quad (4)$$

Finally, let us calculate the intensity with the removal of the second polarizer. Now, if we remove the second polarizer, we only have two polarizers at angles 0° and 67.0° with respect to each other. In the same manner as the previous calculation, unpolarized light passing through the first polarizer is

$$I_1 = \frac{1}{2} I_0. \quad (5)$$

After that, we consider the light passing through the third polarizer. The angle difference between the first and third polarizer is 67.0° , then

$$I_{\text{final}} = I_1 \cos^2(67.0^\circ) = \frac{I_0}{2} \cos^2(67.0^\circ). \quad (6)$$

We can calculate I_{final} by

$$\begin{aligned} I_{\text{final}} &= \frac{I_0}{2} \cos^2(67.0^\circ) \\ &= \left(\frac{\cos^2(67.0^\circ)}{\cos^2(23.0^\circ) \cos^2(44.0^\circ)} \right) \left[\frac{I_0}{2} \cos^2(23.0^\circ) \cos^2(44.0^\circ) \right] \\ &= \left(\frac{\cos(67.0^\circ)}{\cos(23.0^\circ) \cos(44.0^\circ)} \right)^2 I_3 \\ &\simeq 0.35 \times 4.60 \text{ W/cm}^2 \\ &= 1.61 \text{ W/cm}^2. \end{aligned} \quad (7)$$

The intensity of the light after it has passed through the stack, with the second polarizer removed, is approximately 1.61 W/cm^2 .